

Maximum subarray sum problem: history, variations and applications

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Problem statement

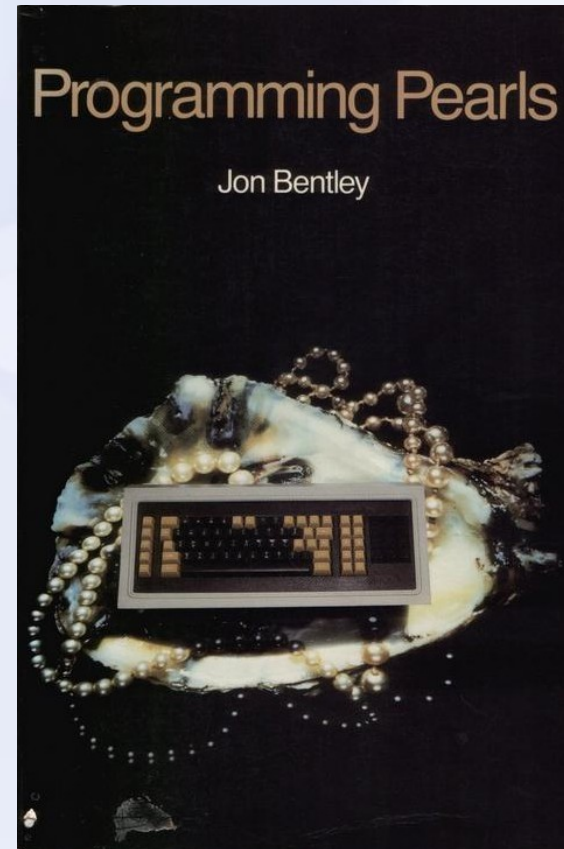
- Input: array of n numbers
- Output: maximum sum in a subarray

Example

- Input: [-1,3,-2,5]
- Output: 6
- Subarrays: [-1], [3], [-2], [5], [-1,3], [3,-2], [-2,5], [-1,3,-2], [3,-2,5], [-1,3,-2,5]
- Optimal choice: [3,-2,5] (sum: 6)

History

- Jon Bentley:
Programming Pearls
(1986)



How to solve it?

- Trivial solution: check all subarrays (time complexity $O(n^2)$)
- However, $O(n)$ is possible

Problem 2

- Input: array of n numbers, range $[a,b]$
- Output: maximum sum of a subarray whose size is between $[a,b]$

Example

- Input: $[-3, 2, 5, -1, 2]$, range $[2, 3]$
- Output: 7
- Optimal choice: $[2, 5]$ (sum: 7)
- [Not allowed: $[2, 5, -1, 2]$ with sum 8]

How to solve it?

- Trivial solution: $O(n^2)$
- Is $O(n \log n)$ possible?
- Or even $O(n)$?

Problem 3

- Input: array of n numbers
- Output: maximum *average* in a subarray
- Example:
 - Input: $[-1, 3, 3, -2, 1]$
 - Output: 3
 - Optimal solution: $[3, 3]$ (sum: 3)
- How to solve it?

Problem 4

- Input: array of n numbers, range $[a,b]$
- Output: maximum average in a subarray whose size is between $[a,b]$
- How to solve it?

Problem 5

- Input: array of n numbers, m operations that change array elements
- Output: after each operation, the maximum subarray sum

Example

- Input: $[-1, 4, -1, 6]$, $[x_2 = -3, x_1 = 8]$
- Output: 6, 10
- Arrays: $[-1, -3, -1, 6]$, $[8, -3, -1, 6]$

How to solve it?

- Trivial: $O(nm)$ (after each operation, calculate the result in $O(n)$ time)
- Any better approaches?

What next?

- Is it possible to combine averages and change operations? (open problem)
- Applications in bioinformatics?



"That's all Folks!"